# Validity of Generalized Second Law of Thermodynamics in the *Logamediate* and *Intermediate* scenarios of the Universe

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In this work, we have investigated the validity of the generalized second law of thermodynamics in *logamediate* and *intermediate* scenarios of the universe bounded by the Hubble, apparent, particle and event horizons using and without using first law of thermodynamics. We have observed that the GSL is valid for Hubble, apparent, particle and event horizons of the universe in the logamediate scenario of the universe using first law and without using first law. Similarly the GSL is valid for all horizons in the intermediate scenario of the universe using first law. Also in the intermediate scenario of the universe, the GSL is valid for Hubble, apparent and particle horizons but it breaks down whenever we consider the universe enveloped by the event horizon.

## I. INTRODUCTION

Nowadays, it is widely accepted fact that the universe is experiencing accelerated expansion driven by dark energy [1] characterized by negative pressure $p_\Lambda$ satisfying the equation of state parameter $w_\Lambda < -1/3$ [2]. Strong observational evidences for dark energy are available from Type Ia supernova (SN Ia), cosmic microwave background radiation and Sloan digital sky survey (SDSS) observations [3] that entities in modern physics. Reviews on dark energy are available in [4]. Although observationally well-established, no single theoretical model provides an eternally compelling framework within which cosmic acceleration or dark energy can provide an entirely compelling framework within which the dark energy can be well-understood. Several models for dark energy have been proposed till date. Such models include quintessence, phantom, quintom, holographic dark energy etc. Discussions on these models are available in Copeland et al [4]. Determining thermodynamic parameters for the expanding (accelerated) universe and verification of the first and the second law for different cosmological horizons, investigating the relation between dynamics and thermodynamics of the universe, studying the conditions required for validity of the generalized second law (GSL) have also been the subjects of interest in recent years [5].

Since the discovery of black hole thermodynamics in 1970s, physicists have been speculating that there should be some relation between black hole thermodynamics and Einstein equations. In 1995, Jacobson [6] derived Einstein equations by applying the first law of thermodynamics $\delta Q = TdS$ together with proportionality of entropy to the horizon area of the black hole. Here $\delta Q$ and $T$ are the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. Verlinde [7] found that the Friedmann equation in a radiation dominated Friedmann-Robertson-Walker (FRW) universe can be written in an analogous form of the Cardy-Verlinde formula, an entropy formula for a conformal field theory. The first law of thermodynamics for the cosmological horizon is given by $-dE = TdS$, where $T = \frac{1}{2\pi l}$ is the Hawking temperature, and $S = \frac{A}{4G}$ is the entropy with $A = 4\pi l^2$ and $G$ as the cosmological horizon area and Newton constant respectively (Cai and Kim in ref [8]). Einstein's field equations have been derived from the first law of thermodynamics in the references given in [8]. In [9], the gravitational field equations for the nonlinear theory of gravity were derived from the first law of thermodynamics by adding some non equilibrium corrections. Profound physical connection between first law of thermodynamics of the apparent horizon and the Friedmann equation was established in [10].

In a spatially flat de Sitter spacetime, the event horizon and the apparent horizon of the Universe coincide and there is only one cosmological horizon. When the apparent horizon and the event horizon of the Universe are different, it was found that the first law and the second law of thermodynamics hold on the apparent horizon, while they break down if one considers the event horizon [11]. There are several studies in thermodynamics for

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dark energy filled universe on apparent and event horizons [12]. Setare and Shafei [13] showed that for the apparent horizon the first law is roughly respected for different epochs while the second law of thermodynamics is respected. Considering the interacting holographic model of dark energy to investigate the validity of the GSL of thermodynamics in a non-flat (closed) universe enclosed by the event horizon, Setare [14] found that generalized second law is respected for the special range of the deceleration parameter. The transition from quintessence to phantom dominated universe was considered and the conditions of the validity of GSL in transition was studied in [15]. In the reference [16], a Chaplygin gas dominated was considered and the GSL was investigated taking into account the existence of the observers event horizon in accelerated universes and it was concluded that for the initial stage of Chaplygin gas dominated expansion, the GSL of gravitational thermodynamics is fulfilled.

In the present work, we study the validity of GSL of thermodynamics in the *intermediate*[18, 19] and *logamediate* [19] expansions of the universe bounded by the Hubble, apparent, particle and event horizons. According to the GSL, for our system, the sum of the entropy of matter enclosed by the horizon and the entropy of the horizon must not be a decreasing function of time. We have investigated the GSL using as well as without using the first law of thermodynamics. While considering the GSL we have taken into account the Hubble horizon, apparent horizon, particle horizon and event horizon.

## II. GENERALIZED SECOND LAW OF THERMODYNAMICS

We consider the Friedmann-Robertson-Walker (FRW) universe with line element

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{1}$$

where $a(t)$ is the scale factor and $k$ is the curvature of the space and $k = 0, 1$ and $-1$ for flat, closed and open universes respectively. The Einstein field equations are given by

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{2}$$

and

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p) \tag{3}$$

where $\rho$ and $p$ are energy density and isotropic pressure respectively and $H = \frac{\dot{a}}{a}$ is the Hubble parameter. The energy conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{4}$$

We denote the radius of cosmological horizon by $R_X$. For Hubble, apparent, particle and event horizon we replace $X$ by $H$, $A$, $P$ and $E$ respectively. The corresponding radii are given by

$$R_H = \frac{1}{H} \; ; \quad R_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} \; ; \quad R_P = a\int_0^t \frac{dt}{a} \; ; \quad R_E = a\int_t^\infty \frac{dt}{a} \tag{5}$$

It can be easily obtained that

$$\dot{R}_H = -\frac{\dot{H}}{H^2} \; ; \quad \dot{R}_A = -HR_A^3\left(\dot{H} - \frac{k}{a^2}\right) \; ; \quad \dot{R}_P = HR_P + 1 \; ; \quad \dot{R}_E = HR_E - 1 \tag{6}$$

To study the generalized second law (GSL) of thermodynamics through the universe we deduce the expression for normal entropy using the Gibb's equation of thermodynamics

$$T_X dS_{IX} = p dV_X + dE_{IX} \tag{7}$$

where, $S_{IX}$ is the internal entropy within the horizon. Here the expression for internal energy can be written as $E_{IX} = \rho V_X$ , where the volume of the sphere is $V_X = \frac{4}{3}\pi R_X^3$. Using equation (7) we obtain the rate of change of internal energy as

$$\dot{S}_{IX} = \frac{4\pi R_X^2}{T_X}(\rho + p)(\dot{R}_X - H R_X) \tag{8}$$

In the following, we shall find out the expressions of the rate of change of total entropy using first law and without using first law of thermodynamics.

## A.  GSL using first law

Issues related to the said horizons in the context of thermodynamical studies are available in the references given in [17]. From the first law of thermodynamics, we have the relation (Ref. Cai and Kim [5])

$$T_X dS_X = 4\pi R_X^3 H(\rho + p) dt \tag{9}$$

where, $T_X$ and $R_X$ are the temperature and radius of the horizons under consideration in the equilibrium thermodynamics.

Using (9) we can get the time derivative of the entropy on the horizon as

$$\dot{S}_X = \frac{4\pi R_X^3 H}{T_X}(\rho + p) \tag{10}$$

Adding equations (8) and (10) we get the time derivative of total entropy as

$$\dot{S}_X + \dot{S}_{IX} = \frac{R_X^2}{G T_X}\left(\frac{k}{a^2} - \dot{H}\right)\dot{R}_X \tag{11}$$

In order the GSL to be hold, we require $\dot{S}_X + \dot{S}_{IX} \geq 0$.

## B.  GSL without using first law

In equation (11) the time derivative of the total entropy is obtained using the first law of thermodynamics. In this paper, we shall also investigate the GSL without using the first law of thermodynamics. The horizon entropy is $S_X = \frac{\pi R_X^2}{G}$ and the temperature is $T_X = \frac{1}{2\pi R_X}$. In this case, the time derivative of the entropy on the horizon is

$$\dot{S}_X = \frac{2\pi R_X \dot{R}_X}{G} \tag{12}$$

Therefore, in this case the time derivative of the total entropy is

$$\dot{S}_X + \dot{S}_{IX} = \frac{2\pi R_X}{G}\left[R_X^2\left(\frac{k}{a^2} - \dot{H}\right)\left(\dot{R}_X - H R_X\right) + \dot{R}_X\right] \tag{13}$$

In the following sections, we shall investigate the nature of the equations (11) and (13) i.e., validity of GS in two scenarios, namely *logamediate* and *intermediate* scenarios.

## III.   GSL IN LOGAMEDIATE SCENARIO

Barrow and Nunes [19] introduced the scenario of 'logamediate' expansion where the scale factor $a(t)$ is given by

$$a(t) = exp(A(\ln t)^\alpha) \ , \quad A\alpha > 0 \ , \quad \alpha > 1 \ , \quad t > 1 \tag{14}$$

Subsequently,

$$H = \frac{A\alpha}{t}(\ln t)^{\alpha-1} \tag{15}$$

and from equation (5), we obtain

$$R_H = \frac{t(\ln t)^{1-\alpha}}{A\alpha} \ ; \quad R_A = \frac{1}{\sqrt{e^{-2A(\ln t)^\alpha}k + \frac{A^2\alpha^2(\ln t)^{-2(1-\alpha)}}{t^2}}} \ ;$$

$$R_P = exp(A(\ln t)^\alpha)\int_0^t \frac{dt}{exp(A(\ln t)^\alpha)} \ ; \quad R_E = exp(A(\ln t)^\alpha)\int_t^\infty \frac{dt}{exp(A(\ln t)^\alpha)} \tag{16}$$

Using (11), (13)-(16) we get the time derivatives of the total entropies to investigate the validity of the GSL in various horizons using and without using first law.

### A.   GSL in the *logamediate* scenario using first law

Here we consider the GSL in the *logamediate* scenario using the first law of thermodynamics. Using (11) and (16) we get the time derivative of total entropies as follows:

- For Hubble horizon

$$\dot{S}_H + \dot{S}_{IH} = \frac{e^{-2A(\ln t)^\alpha}(\alpha - 1 - \ln t)(\ln t)^{-3\alpha}\left(-kt^2(\ln t)^2 + A\alpha e^{2A(\ln t)^\alpha}(\ln t)^\alpha(\alpha - 1 - \ln t)\right)}{A^3\alpha^3 G T_H} \tag{17}$$

- For apparent horizon

$$\dot{S}_A + \dot{S}_{IA} = \frac{A\alpha e^{A(\ln t)^\alpha}(\ln t)^\alpha\left(kt^2(\ln t)^2 - Ae^{2A(\ln t)^\alpha}\alpha(\alpha - 1 - \ln t)(\ln t)^\alpha\right)^2}{G T_A\left(kt^2(\ln t)^2 + A^2 e^{2A(\ln t)^\alpha}\alpha^2(\ln t)^{2\alpha}\right)^{5/2}} \tag{18}$$

- For particle horizon

$$\dot{S}_P + \dot{S}_{IP} = \frac{e^{-2A(\ln t)^\alpha}R_P^2\left(t\ln t + A\alpha(\ln t)^\alpha R_P\right)\left(kt^2(\ln t)^2 - Ae^{2A(\ln t)^\alpha}\alpha(\alpha - 1 - \ln t)(\ln t)^\alpha\right)}{t^3(\ln t)^3 G T_P} \tag{19}$$

- For event horizon

$$\dot{S}_E + \dot{S}_{IE} = \frac{e^{-2A(\ln t)^\alpha}R_E^2\left(-t\ln t + A\alpha(\ln t)^\alpha R_E\right)\left(kt^2(\ln t)^2 - Ae^{2A(\ln t)^\alpha}\alpha(\alpha - 1 - \ln t)(\ln t)^\alpha\right)}{t^3(\ln t)^3 G T_E} \tag{20}$$

In figures 1-4 we have investigated the GSL using the first law of thermodynamics. Here also the time derivative of the horizon entropy is calculated using the first law of thermodynamics. In figures 1 and 2 we have plotted the time derivatives of the total entropies against the cosmic time $t$ for the Hubble and apparent horizons in the *logamediate* scenario. We have computed the total entropy using the first law of thermodynamics. We see that the rate of change of total entropies are decreasing with time. We find that
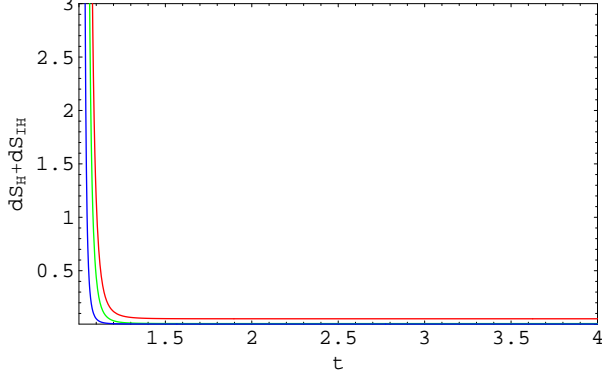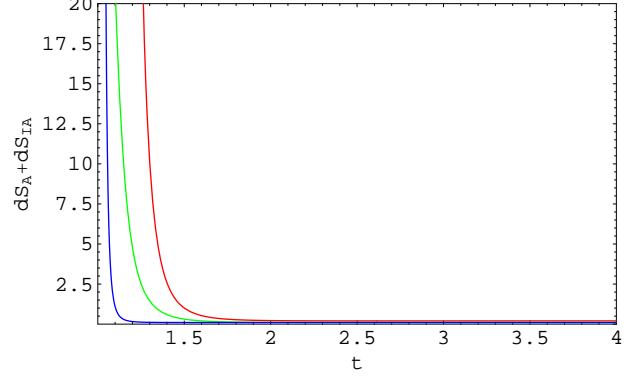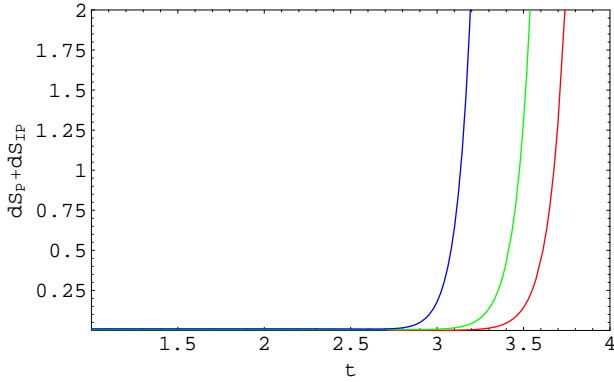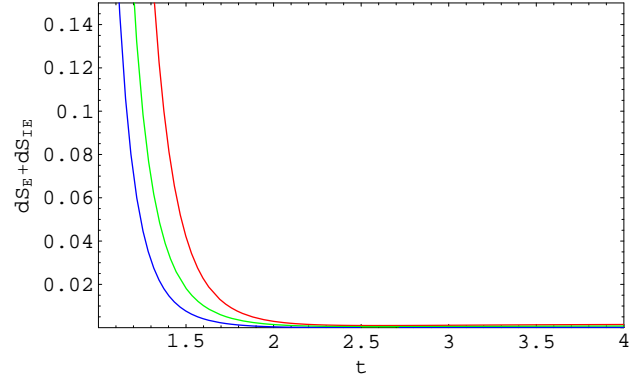
Fig.1



Fig.2



Fig.3



Fig.4

Figs. 1, 2, 3 and 4 show the time derivatives of the total entropy for Hubble horizon $R_H$, apparent horizon $R_A$, particle horizon $R_P$ and event horizon $R_E$ respectively **using first law of thermodynamics** in the *logamediate* scenario. The red, green and blue lines represent the $dS_X + dS_{IX}$ for $k = -1$, 1 and 0 respectively. We have chosen $A = 5$, $\alpha = 2$.

$\dot{S}_X + \dot{S}_{IX} > 0$ throughout the evolution of the universe. Thus, the GSL is valid in both of the Hubble and apparent horizons.

In figure 3, we have plotted the time derivative of the total entropy assuming particle horizon as the enveloping horizon of the universe. We see that the rate of change of total entropy is increasing with time. In this figure the time derivative stays at the positive level. This indicates that the GSL is valid on the particle horizon. Also, in figure 4, where the time derivative of the total entropy is plotted for the universe enveloped by the event horizon, we see that the rate of change of total entropy is decreasing with time. We find that $\dot{S}_E + \dot{S}_{IE} > 0$ throughout the evolution of the universe. This indicates that when we consider the cosmological event horizon as the enveloping horizon of the universe, the GSL is also valid. So we conclude that the GSL is always valid for Hubble, apparent, particle and event horizons in the *logamediate* scenario of the universe when we have calculated the horizon entropies using first law of thermodynamics.

### B. GSL in the *logamediate* scenario without using first law

Without using the first law, the time derivative of the total entropies in the *logamediate* scenario come out as
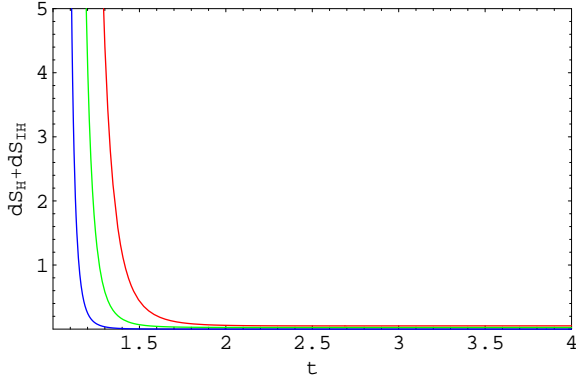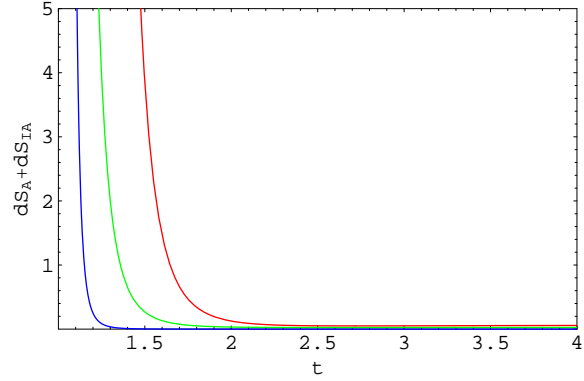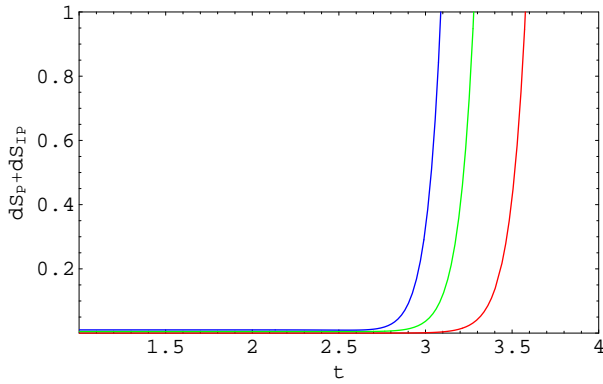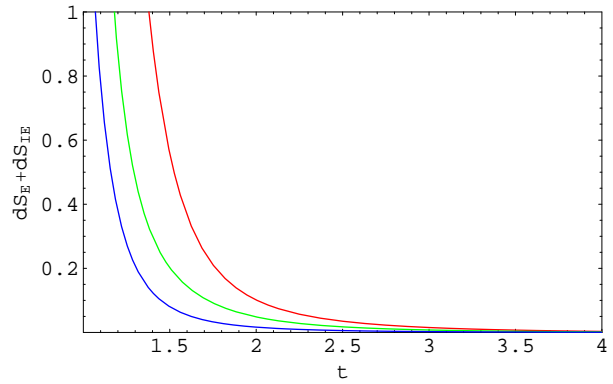
Fig.5

Fig.6

Fig.7

Fig.8

Figs. 5, 6, 7 and 8 show the time derivatives of the total entropy for Hubble horizon $R_H$, apparent horizon $R_A$, particle horizon $R_P$ and event horizon $R_E$ respectively **without using first law of thermodynamics** in the *logamediate* scenario. The red, green and blue lines represent the $dS_X + dS_{IX}$ for $k = -1$, $1$ and $0$ respectively. We have chosen $A = 5$, $\alpha = 2$.

- For Hubble horizon

$$\dot{S}_H + \dot{S}_{IH} = \frac{2e^{-2A(\ln t)^\alpha} \pi t (\ln t)^{1-4\alpha} \left(kt^2(\ln t)^2(1 - \alpha + \ln t) + A\alpha(\ln t)^\alpha(-kt^2(\ln t)^2 + e^{2A(\ln t)^\alpha}(1 - \alpha + \ln t)^2)\right)}{A^4\alpha^4 G}$$

(21)

- For apparent horizon

$$\dot{S}_A + \dot{S}_{IA} = \frac{2Ae^{2A(\ln t)^\alpha} \pi t \alpha(\ln t)^{1+\alpha} \left(kt^2(\ln t)^2 - Ae^{2A(\ln t)^\alpha}\alpha(\alpha - 1 - \ln t)(\ln t)^\alpha\right)^2}{G \left(kt^2(\ln t)^2 + A^2\alpha^2 e^{2A(\ln t)^\alpha}(\ln t)^{2\alpha}\right)^3}$$

(22)

- For particle horizon

$$\dot{S}_P + \dot{S}_{IP} = \frac{2\pi R_P^3}{G} \left(ke^{-2A(\ln t)^\alpha} - \frac{A\alpha(\alpha - 1)(\ln t)^{-2+\alpha}}{t^2} + \frac{A\alpha(\ln t)^{\alpha-1}}{t^2}\right) + \frac{2\pi R_P}{G} \left(\frac{A\alpha R_P(\ln t)^{\alpha-1}}{t} + 1\right)$$

(23)

- For event horizon

$$\dot{S}_E + \dot{S}_{IE} = -\frac{2\pi R_E^3}{G} \left(ke^{-2A(\ln t)^\alpha} - \frac{A\alpha(\alpha - 1)(\ln t)^{-2+\alpha}}{t^2} + \frac{A\alpha(\ln t)^{\alpha-1}}{t^2}\right) + \frac{2\pi R_E}{G} \left(\frac{A\alpha R_E(\ln t)^{\alpha-1}}{t} - 1\right)$$

(24)

In figures 5 - 8 we have plotted $\dot{S}_X + \dot{S}_{IX}$ based on equations (21) - (24) against cosmic time $t$ without using the first law of thermodynamics in the *logamediate* scenario. Here also we find similar results to those obtained using the first law of thermodynamics. From figures 5-8 we find that the GSL is valid in the *logamediate* scenario without using the first law for the universe enveloped by the Hubble, apparent, particle and event horizons.

## IV.   GSL IN INTERMEDIATE SCENARIO

Barrow and Liddle [18] proposed a model of 'intermediate' expansion, where the scale factor is given by

$$a(t) = exp(Bt^\beta) , \quad B > 0, \ \ 0 < \beta < 1 \tag{25}$$

This model bears many qualitative similarities to power-law inflation: like power-law inflation, there is no natural end to inflation and a mechanism must be introduced in order to bring inflation to an end. Also, as with power-law inflation, intermediate inflation offers the possibility of density perturbation and gravitational wave spectra which differ significantly from the usual inflationary prediction of a nearly flat spectrum with negligible gravitational waves [18]. Using the above form of scale factor, we get

$$H = B\beta t^{\beta-1} \tag{26}$$

Subsequently from equation (5), we obtain,

$$R_H = \frac{t^{1-\beta}}{B\beta} \ ; \ \ R_A = \frac{1}{\sqrt{ke^{-2Bt^\beta} + B^2\beta^2 t^{2(\beta-1)}}} \ ;$$

$$R_P = \frac{B^{-\frac{1}{\beta}} e^{Bt^\beta} \left(\Gamma\left[\frac{1}{\beta}\right] - \Gamma\left[\frac{1}{\beta}, Bt^\beta\right]\right)}{\beta} \ ; \ \ R_E = \frac{B^{-\frac{1}{\beta}} e^{Bt^\beta} \Gamma\left[\frac{1}{\beta}, Bt^\beta\right]}{\beta} \tag{27}$$

Using the above expressions in equations (11) and (13) we can get the time derivatives of the total entropies using as well as without using the first law of thermodynamics.

### A.   GSL in the *intermediate* scenario using first law

In this subsection we consider the GSL in the *intermediate* scenario. Using the first law the time derivatives of the total entropies are

- For Hubble horizon

$$\dot{S}_H + \dot{S}_{IH} = \frac{e^{-2Bt^\beta} t^{-3\beta}(\beta - 1)\left(-kt^2 + Be^{2Bt^\beta} t^\beta(\beta-1)\beta\right)}{B^3\beta^3 GT_H} \tag{28}$$

- For apparent horizon

$$\dot{S}_A + \dot{S}_{IA} = \frac{B\beta e^{Bt^\beta} t^\beta \left(kt^2 - Be^{2Bt^\beta} t^\beta(\beta-1)\beta\right)^2}{GT_A \left(kt^2 + B^2 e^{2Bt^\beta} t^{2\beta}\beta^2\right)^{5/2}} \tag{29}$$

- For particle horizon

$$\dot{S}_P + \dot{S}_{IP} = \frac{B^{-\frac{3}{\beta}} \left(\Gamma\left[\frac{1}{\beta}\right] - \Gamma\left[\frac{1}{\beta}, Bt^\beta\right]\right)^2 \left(kt^2 - Be^{2Bt^\beta} t^\beta(\beta-1)\beta\right)\left(B^{\frac{1}{\beta}}t + Be^{Bt^\beta} t^\beta \left(\Gamma\left[\frac{1}{\beta}\right] - \Gamma\left[\frac{1}{\beta}, Bt^\beta\right]\right)\right)}{t^3\beta^2 GT_P} \tag{30}$$
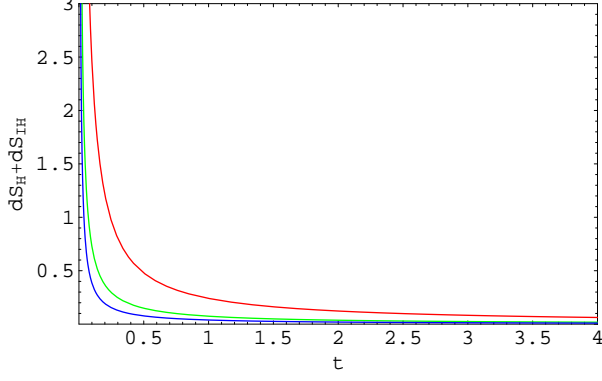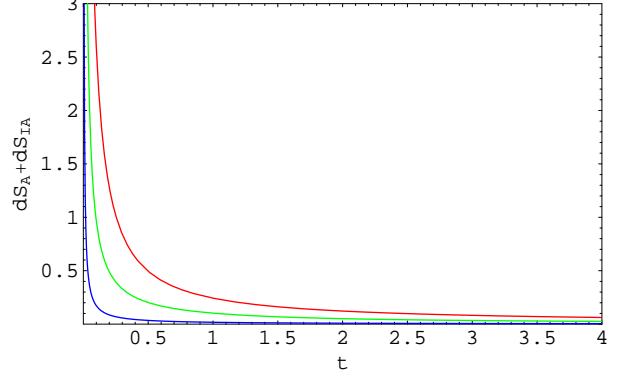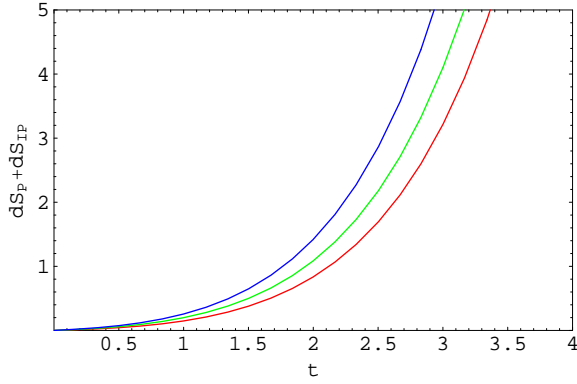
Fig.9



Fig.10



Fig.11
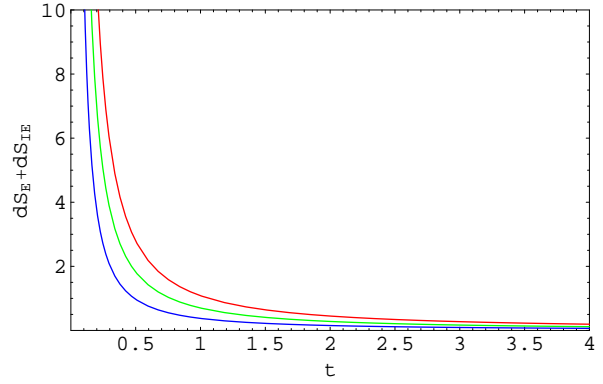


Fig.12

Figs. 9, 10, 11 and 12 show the time derivatives of the total entropy for Hubble horizon $R_H$, apparent horizon $R_A$, particle horizon $R_P$ and event horizon $R_E$ respectively **using first law of thermodynamics** in the *intermediate* scenario. The red, green and blue lines represent the $dS_X + dS_{IX}$ for $k = -1$, 1 and 0 respectively. We have chosen $A = 5$, $\alpha = 2$.

- For event horizon

$$\dot{S}_E + \dot{S}_{IE} = \frac{B^{-\frac{3}{\beta}} \left( \Gamma \left[ \frac{1}{\beta}, Bt^\beta \right] \right)^2 \left( kt^2 - Be^{2Bt^\beta} t^\beta (\beta - 1)\beta \right) \left( -B^{\frac{1}{\beta}} t + Be^{Bt^\beta} t^\beta \Gamma \left[ \frac{1}{\beta}, Bt^\beta \right] \right)}{t^3 \beta^2 G T_E} \quad (31)$$

In figures 9 - 12 we have investigated the GSL for the *intermediate* scenario using the first law of thermodynamics using equations (28) - (31). Like the *logamediate* scenario, the GSL in this situation is valid for the universes enveloped by Hubble, apparent and event horizon. Also, in all these cases, the time derivatives of the total entropy are falling with the passage of cosmic time. However, in the case of the universe enveloped by the apparent horizon, the time derivative of the total entropy is positive and increasing throughout the evolution of the universe. In this case also the GSL is satisfied. So we may conclude that the GSL is valid for all horizons in intermediate scenario of the universe using first law.

## B. GSL in the *intermediate* scenario without using first law

The time derivatives of the total entropies are also calculated without using the first law of thermodynamics in the *intermediate* scenario as follows:
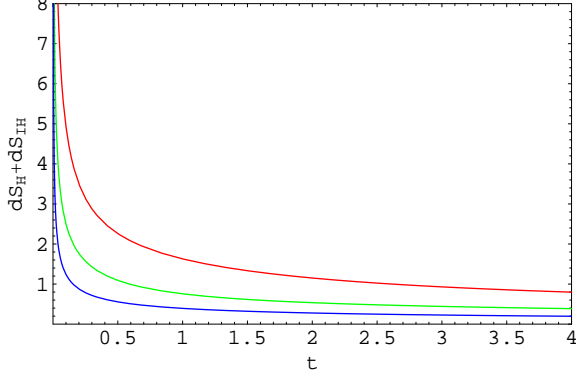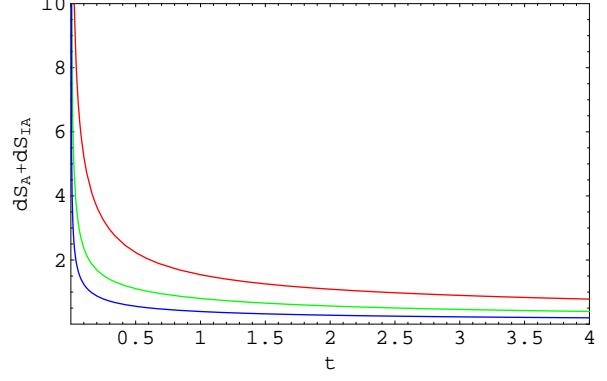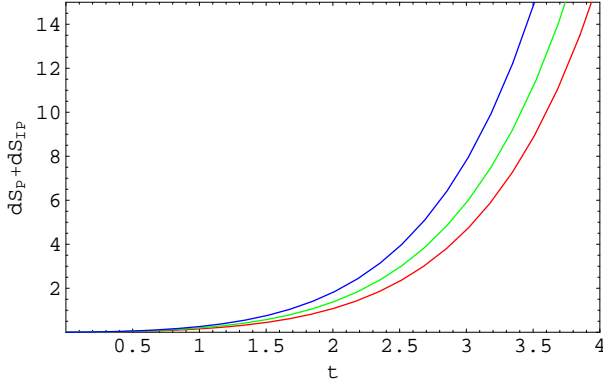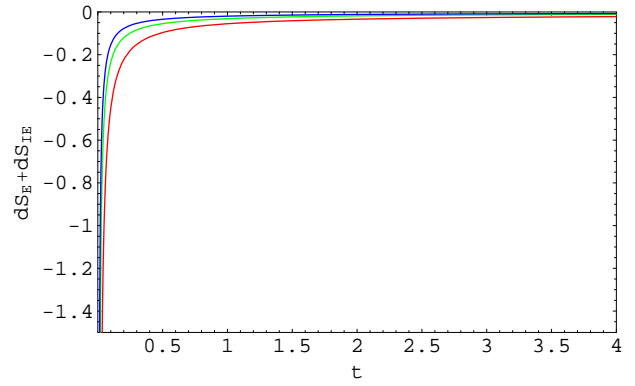
Fig.13



Fig.14



Fig.15



Fig.16

Figs. 13, 14, 15 and 16 show the time derivatives of the total entropy for Hubble horizon $R_H$, apparent horizon $R_A$, particle horizon $R_P$ and event horizon $R_E$ respectively **without using first law of thermodynamics** in the *intermediate* scenario. The red, green and blue lines represent the $dS_X + dS_{IX}$ for $k = -1, \ 1$ and $0$ respectively. We have chosen $A = 5, \ \ \alpha = 2$.

- For Hubble horizon

$$\dot{S}_H + \dot{S}_{IH} = \frac{2\pi e^{-2Bt^\beta} t^{1-4\beta} \left( Be^{2Bt^\beta} t^\beta \beta (\beta - 1)^2 - kt^2(\beta - 1(1 + Bt^\beta)) \right)}{B^4 \beta^4 G} \tag{32}$$

- For apparent horizon

$$\dot{S}_A + \dot{S}_{IA} = \frac{2\pi\beta Be^{2Bt^\beta} t^{1+\beta} \left( kt^2 - Be^{2Bt^\beta} t^\beta (\beta - 1)\beta \right)^2}{G \left( kt^2 + B^2\beta^2 e^{2Bt^\beta} t^{2\beta} \right)^3} \tag{33}$$

- For particle horizon

$$\dot{S}_P + \dot{S}_{IP} = \frac{2\pi B^{-\frac{1}{\beta}} e^{Bt^\beta} \left( \Gamma\left[\frac{1}{\beta}\right] - \Gamma\left[\frac{1}{\beta}, Bt^\beta\right] \right)}{\beta^3 G} \times \left[ \beta^2 + B^{1-\frac{1}{\beta}} \beta^2 e^{Bt^\beta} t^{\beta-1} \left( \Gamma\left[\frac{1}{\beta}\right] - \Gamma\left[\frac{1}{\beta}, Bt^\beta\right] \right) \right.$$

$$\left. + B^{-\frac{2}{\beta}} \left( k - Be^{2Bt^\beta} t^{\beta-2}(\beta - 1)\beta \left( \Gamma\left[\frac{1}{\beta}\right] - \Gamma\left[\frac{1}{\beta}, Bt^\beta\right] \right) \right)^2 \right] \tag{34}$$

- For event horizon

$$\dot{S}_E + \dot{S}_{IE} = \frac{2\pi B^{-\frac{3}{\beta}} e^{Bt^\beta} \Gamma\left[\frac{1}{\beta}, Bt^\beta\right]}{t^2 \beta^3 G} \times \left[\beta B e^{Bt^\beta} t^\beta \left(\beta B^{\frac{1}{\beta}} t + (\beta - 1)e^{Bt^\beta} \Gamma\left[\frac{1}{\beta}, Bt^\beta\right]\right)\right.$$

$$\left. -t^2 \left(\beta^2 B^{\frac{2}{\beta}} + k \left(\Gamma\left[\frac{1}{\beta}, Bt^\beta\right]\right)^2\right)\right] \tag{35}$$

Time derivatives of the total entropies are calculated based on the equations (32) - (35) and are plotted against the cosmic time $t$. Figures 13 - 15 show the staying of the time derivative of the total entropy at positive level throughout the evolution of the universe. This indicates the validity of the GSL in the universes enveloped by Hubble, apparent and particle horizons. However, it fails to stay at positive level in the case of the universe enveloped by the event horizon. This indicates the breaking down of the GSL without using the first law of thermodynamics in the *intermediate* scenario.

## V. DISCUSSIONS

In the present work, our endeavor was to investigate the validity of the generalized second law of thermodynamics in the *logamediate* and *intermediate* scenarios of the universe bounded by the Hubble, apparent, particle and event horizons. We have investigated the generalized second law using two different approaches namely, (i) using the first law of thermodynamics and (ii) without using the first law of thermodynamics.

As previously stated, the basic aim is to investigate whether $dS_X + dS_{IX}$ i.e. the time derivative of the sum of the entropy of the universe bounded by the horizons and the normal entropy remains at non-negative level. To do the same, we have considered the universe enveloped by Hubble, apparent, particle and event horizons. The logamediate as well as intermediate scenarios have been investigated for the nature of the time derivative of the total entropy for four of the said horizons. With suitable choice of the model parameters, we have seen from figures 1 - 4 that if we use the first law to derive the time derivative of the total entropy, the generalized second law is valid for all of the four horizons irrespective of the curvature of the universe. Similar situation happens (see figures 5 - 8) when we ignore the first law to get the time derivative of the total entropy. While considering the intermediate scenario we find from figures 9 - 12 that the generalized second law based on first law is always valid for all types of enveloping horizons and the curvature of the universe. However, if we ignore the first law, we find that in intermediate scenario the generalized second law breaks down (see figure 16) for open, flat and closed universe enveloped by the event horizon. The generalized second law without the first law in this scenario is valid if the enveloping horizons are Hubble, apparent and particle horizons (see figures 12 - 15). The validity of the generalized second law occurs irrespective of the use of the first law of thermodynamics in calculating the time derivative of the total entropy.

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